Shill-Proof Fee (SPF) Schedule: the Sunscreen against Seller Self-Collusion in Online English Auctions

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Shill bidding in English auctions is the use of insincere bids on the seller's behalf to artificially drive up the price of the listing. Shilling is illegal and has become a serious problem in online auctions because it is easy for the seller to bid under false names and hence self-collude. We show that in an independent private-value (IPV) English auction where there are heterogeneous bidders the seller can increase her expected profit by starting with a low ex ante reserve price and then optionally resetting it via shilling after observing the second highest bidder's valuation. A shill-proof mechanism needs to ensure that the seller's optimal strategy is to set her optimal reserve based upon her best estimation of the possible presence of a high-type bidder ex ante and not to revise the reserve via shilling. We introduce a Shill-Proof Fee (SPF) schedule for IPV English auctions where an auctioneer charges the seller a commission fee not based upon the final sale price but the difference between the final sale price and the seller's disclosed reserve. Commission rates vary across auctions, and are mathematically determined to guarantee the non-profitability of shilling.

Keywords: *Auction; Buyer Behavior; E-commerce; Game Theory; Internet Marketing; Market Structure; Selling*

I. Introduction

"eBay can be a fun place for buying and selling but… it's also full of cons, crumbs and kooks" (Anonymous). Nearly one-third of online Americans have participated in Internet auctions, 80% of them have purchased an item through auctions and nearly one-third of participants have sold items. But unfortunately, 41% of all buyers acknowledged having a problem at one time or another (Internet Prophets 2004). Among the thousands of consumer fraud complaints the Federal Trade Commission (FTC) receives yearly, those dealing with online auction fraud consistently rank at or near the top of the list[1] (Federal Trade Commission 2004). *Shill bidding* or *shilling* – the use of insincere bids on the seller's behalf to artificially drive up the price – has been identified as one of the most common types of Internet auction fraud (Internet Fraud Complaint Center 2001)[2]. High-profile shilling cases have even become headline news[3]. For instance, in 2001, three men were indicted for operating a shilling ring that auctioned a fake Richard Diebenkorn painting for $135,805 on eBay (U.S. Department of Justice 2001). One of them, accused of inflating bids in hundreds of online art auctions and netting $450,000+ over a 20-month period, has received a 4-year prison term and a heavy fine (Lee 2004). Anecdotes of shilling can also be found on several websites[4].

Lift-lining in live English auctions is similar to shilling. It refers to a common practice of auctioneers to use phantom bids to push up the final price (Cassady 1967). Auctioneers have the incentives to lift-line in live auctions because they can increase their commission fees and attract future sellers by extracting higher sale prices. However, shilling, initiated by the seller but not supported by the auctioneer, is rare, costly and difficult to achieve in live auctions. The seller cannot simply appear on the auction floor to shill bid and therefore has to hire associates. She has to overcome the principal-agent problem ensuring that her associates truly act on her behalf and are capable of executing strategies that maximize her benefits. Moreover, the seller needs to deploy signalling and punishment strategies for her associates as well as schemes on how to divide the spoils.

In comparison, Internet auctioneers do not have strong incentives to lift-line as auctioneers in live auctions because they charge much lower commission rates and handle much larger volumes of transactions. But the Internet provides the perfect environment for the seller to shill; shilling is cheap, easy and hard to detect in online auctions. Creating new email addresses and aliases is almost costless and nearly effortless, allowing the seller to self-collude. The lack of the strong binding between a screen-name and a real identity – the intrinsic problem of the Internet – provides unprecedented opportunities for identity misrepresentation and fraudulent bids. In the indictment case discussed above, the defendants created 40+ IDs using false registration information and used these aliases to shill. They even created and utilized IDs containing the last names of the renowned artists to trick buyers into believing that family members of the artists were bidding. With the easy adoption of multiple aliases online, the seller can simply shill all by herself; there are no principal-agent problems and she can retain all the additional economic rents.

Auction fraud is on the rise, yet there is a lack of theoretical studies. Klemperer (2002) has cautioned of the danger of such thinness, stating that most auction literature assumes a fixed number of bidders who behave non-cooperatively and that auction theorists pay little attention to the most important issue in auction design – how to discourage collusive and predatory behavior. In the limited literature on collusion, most research studies how bidders collude to either deter the entry or depress the bidding of rivals in order to avoid bidding up prices (McAfee and McMillan 1987, McAfee and McMillan 1992, Milgrom and Weber 1982, Milgrom 2000, Graham and Marshall 1987, Klemperer 2002, Fabra 2003). Theoretical studies on seller self-collusion, i.e., shilling, are even more limited (Graham, Marshall and Richard 1990, Bag, Dinlersoz and Wang 2000, Sinha and Greenleaf 2000, Chakraborty and Kosmopoulou 2004).

Graham et al. (1990) show that when private-value bidders draw their valuations from heterogeneous probability distributions, the auctioneer can increase the expected seller profit by dynamically resetting the reserve price upon learning the valuation of the second highest bidder. Their result is based on the strong assumption that the auctioneer knows not only each of these value distributions but also the exact number of bidders in each distribution. Bag et al. (2000) prove that shilling can be profitable for the seller where bidders draw their private valuations from one of two distributions with disjoint supports. Sinha and Greenleaf (2000) study auctions where bidding is restricted to a number of exogenously-set bid levels with significant gaps between possible bids and conclude that discrete bidding combined with differences in bidders' aggressiveness can lead to profitable shilling. Chakraborty and Kosmopoulou (2004) show that shilling in common-value auctions reduces both the seller's and bidders' expected profits and the only party who gains is the auctioneer.

None of the previous research offers a solution to deter or prevent shill bidding. Vickrey (1961) briefly suggested the use of a trustworthy bid-holder as a means to prevent shilling in a sealed-bid second price auction: "To prevent the use of a 'shill' to jack the price up by putting in a late bid just under the top bid, it would probably be desirable to have all bids delivered to

[1]Being consistent with the FTC data, auction fraud has topped Internet fraud for five years in a row 1998-2002 according to the Internet Fraud Watch operated by the National Consumers League: in 1998, online auction fraud accounted for 68% of reported Internet incidents. In 1999, 87%. In 2000, 78%. In 2001, 70%. And in 2002, a soaring 90%. Data on year 2003 and 2004 are not available.

[2]IFCC stands for Internet Fraud Complaint Center, which is a partnership between the Federal Bureau of Investigation (FBI) and the National White Collar Crime Center (NW3C).

[3]Examples of shilling were reported in the New York Times and the Wall Street Journal (Dobrzynski 2000, Simpson 2000, Schwartz and Dobrzynski 2001).

[4]Here are some of the websites describing details of suspicious shill bidding: http://www.geocities.com/ebayoyvey/ebayshills.html and http://www.geektimes.com/michael/culture/crime/ebay/shill.html.

and certified by a trustworthy holder, who would then deliver all bids simultaneously to the seller." Unfortunately, there has been no follow-up study on this issue in the past four decades.

Our present research extends the previous results on the optimal reserve and shilling strategies and, more importantly, introduces an intermediation fee schedule to prevent shilling. We study these issues in the context of the independent private-value (IPV) model of single-round English auctions with continuous bidding. We also study heterogeneous bidders, but with a more relaxed assumption that these bidders can have possibly overlapping value distributions and the auctioneer and the seller know the probability that a bidder has a certain type, but do not need to know exactly how many bidders of each type are present.

The rationale for the profitability of shilling is as follows. The seller, as bidders often do (Cramton 1998), can update their "bidding" strategies upon learning from the bidding process in an open auction. She can use shill bids as costless (or low cost) threats to corner high-value bidders. Shilling can be profitable especially if the seller's expected profit function has multiple local maxima caused by different types of bidders, such as collectors vs. dealers or bidding for winning vs. bidding for participation. The multiple local maxima motivate a seller to start with a reserve price corresponding to a low local maximum of her expected profit and then to maximize her expected profit by optionally shilling to reset her reserve corresponding to a higher local maximum upon observing the second highest bidder's valuation.

The possibility of shilling greatly reduces the pressure on the seller to set the optimal reserve price ex ante. She can start low and then shill to reset it. The lower the disclosed reserve price, the more attractive the auction, which provides another incentive for shilling. Besides, the ascending structure of an English auction allows a cheat time for the seller to assess the success of her shilling strategy. Consequently, a bidder's strategic response to shilling is to snipe – to delay his bid until the last minutes – in order to avoid disclosing information and shorten the seller's cheat time. The substantial shares of late bids in Internet auctions, and the fact that the amount of late bidding in eBay-type hard-close auctions is much more than that of Amazon-type soft-close auctions[5] (Roth and Ockenfels 2002) may reflect the bidders' consideration of shilling.

Note that it is always the seller who plots behind the scenes and the ultimate goal of shilling is to benefit the seller. Hence any economic mechanism or policy against shilling requires a redesign of the seller's incentives. Since the only imminent major party that can control the seller's incentive is the auctioneer, we aim to revise the auctioneer's policies towards the seller to eliminate, or at least reduce, the seller's incentives to shill.

An auctioneer has two major controls over a seller: whether or not to allow a seller to list her item, and what intermediation fees a seller is charged. Since there is no way of knowing whether a seller will shill or not before an auction starts, an auctioneer cannot deter shilling by restricting entries. But an auctioneer can use his fee schedule to control the seller's shilling strategy[6]. As we have mentioned, shilling in an IPV English auction is equivalent to resetting the reserve price during the auction, therefore we aim to design a Shill-Proof Fee (SPF) schedule where setting an ex ante optimal reserve price and not to shill is the best strategy for the seller.

Currently, most auctioneers have similar intermediation fee structures: an auctioneer charges a seller a listing fee, based on the reserve price, before an auction starts, and a commission fee, a percentage of the final price, if the auction is ended with a sale. Such fee structures are not designed to prevent shilling, and may even actually encourage such a fraud. For instance, eBay charges a low listing fee and a commission rate uniform in all markets except for motor vehicles and real estate[7]. As of June 2004, for a regular non-featured reserve-price English auction, eBay's listing fee is between \$0.30 and \$4.80, ascending with the reserve. A seller is motivated to start with a low reserve price and shill because she can at least save on the listing fee (Kauffman and Wood 2005). eBay's commission rates are not designed to deter shilling either. They are 5.25% for the initial \$25 of the closing value, plus 2.75% for \$25.01 - \$1000, and 1.50% for the remaining closing value balance (\$1000.01 - closing value). Suppose an auction currently has the highest bid of \$10,000 and the seller chooses to shill bid at \$10,100 (\$100 is the minimum increment set by eBay for bids above \$5000). If the shill bid succeeds and is outbid, say by \$10,200, the seller will have an additional net profit of $\$197 = \$200 - \$3$. If the shill bid is not outbid, the seller needs to pay the auctioneer \$164.62, and loses the sure deal that she has had. The seller is better off shilling if she estimates that there is at least an 87% chance that her shill bid will succeed (assume her valuation is \$8,700), which is quite possible considering that a bidder already willing to pay \$10,000 is unlikely to walk away because of a mere \$200 price increase. The seller has an incentive to shill as long as she can increase her expected profit. Moreover, eBay's policy of relisting does not discourage shilling either; if an item does not sell either because of no bid above the reserve or a non-paying winning bidder (who may be a shilling seller), eBay allows a full or partial refund of the "final value fee credit." This means that after a period of time, the seller who fails to sell because of shilling can have a second round auction at no charge.

How to set intermediation fees that prevent shill bidding? An auctioneer should not set the them too low because a seller will have the incentive to shill as long as it leads to an expected increase of the sale price that is greater than the expected increase of intermediation fees. On the other hand, the auctioneer should

[5] An Amazon-type soft-close auction refer to the auction whose deadline is automatically extended to a certain period of time after the last submitted bid. Auctions at uBid.com also have soft-close deadlines. In Yahoo! auctions, the seller has an option to label her auction with fixed-close time or "auto-extension."

[6] It has been shown that fee schedules charged by auctioneers do have an impact on both the sellers' choice of auctions and the success rate of resulting sales (Lucking-Reiley 2000). For instance, Yahoo! used to have no listing fee and the lowest success rate (16%). But eBay has relatively high listing fees among online auction sites and high success rates (54%).

[7] For details, please go to http://pages.ebay.com/help/sell/fees.html.

not set fees too high because a seller would then go to competing auctioneers who charge less. Keeping these factors in mind, we design an SPF schedule, under which the auctioneer charges a seller the intermediation fees according to the following: 1) a listing fee which is a function of the seller's disclosed reserve; and 2) a commission fee which is a function of a commission rate and the difference between the final sale price and the disclosed reserve. Commission rates are mathematically determined to ensure the non-profitability of shilling: shilling will not provide extra expected profit to the seller, and she is rewarded with a low commission fee if she has disclosed a reserve price close to her expected final sale price. Commission rates under the SPF may differ across auctions because they are a function of bidders' value distributions. By contrast, most auctioneers use one or few flat commission rates for all auctions and there is no theory of how these rates are determined and why they are good choices. Our design of variable rates has theoretical support and the conclusion conforms to what is suggested by Klemperer (2002) that: "Auction design is not 'one size fits all' ... In the practical design of auctions, local circumstances matter and the devil is in the details."

The most obvious benefit of shilling, also the focus of this paper, is to push up the sale price. But shilling can deliver other benefits to the seller as well: 1) she can reduce her listing fee by starting with a low reserve and resetting it with shill bids early in an auction (Kauffman and Wood 2005); 2) if nobody has bid yet, the seller can shill to jump-start the bidding. Shilling early in an auction has a marketing effect and it is well known that the more the bidders, the better off the seller (Bulow and Klemperer 1996); and 3) shilling can create for the seller an opportunity to leave positive feedback on herself in case her shill bid is the final winning bid. In the above indictment case (U.S. Department of Justice 2001), the ring members left themselves great feedbacks like "Painting Way Better Than Expected. You Can Do Business Here With Confidence!!!!!" Therefore, any mechanism or policy preventing or deterring shilling will also enhance the credibility of the reputation systems. Unfortunately, current studies on Internet auction feedback systems (Dellarocas 2003, Resnick and Zeckhauser 2002) fail to incorporate the effects of shilling.

The paper proceeds as follows. In Section II, we explain why current methods against shilling are not effective. In Section III, we theoretically analyze why shilling can provide the seller extra expected profits in an IPV English auction and analyze how the seller determines her optimal shill bid level. In Section IV, we prove why our SPF schedule prevents shilling, discuss the practicality of applying the SPF, and analyze the seller's strategies. Section V concludes our analysis.

II. Why Are the Current Methods against Shilling Not Effective?

Without proper controls, shilling will continue and may increase, damaging consumers' trust in online auction markets. Not surprisingly, because market participants have gradually realized the adverse effects of shilling[8], several methods – building a fraud-conscious community, data mining auction records, and verifying digital identities – have been suggested to deter auction fraud, but all of these methods are limited in their applicability and scope.

Limitations of Building a Fraud-conscious Community From a social engineering perspective, many auctioneers, law enforcement agencies, relevant business partners, and volunteers provide tips to raise awareness of auction fraud. In 2000, the FTC launched its SafeBid project to train, educate and coordinate law enforcement agencies to detect and prosecute Internet auction fraud, including shilling. In 2004, the FTC published tips for buyers and sellers of Internet auctions. These efforts are helpful to build a fraud-conscious culture, but there is still an urgent need for more effective deterrents. At present, the most common deterrent in practice is to detect evidence of fraud via data mining.

Limitations of Data Mining Auction Records eBay's current strategy is to rely on user reports to trigger investigations of shilling and then study statistics on the bidding records to discover evidence. A seller is temporarily suspended the first time shilling is discovered, and is permanently suspended with reoccurring offenses. A major challenge of this approach is how to define the patterns of shilling. It is difficult to tell whether or not a seller has shilled from the observed behavior in the auction. For instance, if a bidder exclusively bids on a particular seller's items, it doesn't mean the bidder is a shilling seller, he may do so just because he only trusts this seller. eBay considers the patterns of shilling are those "that suggest no advantage to the bidder but significantly increase the bidding price of the listing." Although eBay provides examples of questionable patterns[9], it is infeasible to define all possibilities because experienced shill bidders can always discover new ways to outsmart these patterns. Even if profiles of shills were well-defined, statistical methods would still be costly because an auctioneer needs to analyze detailed bidding records over a long period of time in order to find correlations, and thorough investigation is required to prosecute a shill. Hence the chance for a shill to be caught through the current statistical methods is very slim and an auctioneer rarely punishes a seller even if suspicious behavior is reported. Besides, how to prevent a suspended seller from adopting another alias given the ease of switching online identities? Therefore, other methods are sought and many have suggested methods of verifying digital identities to solve the root of the problem – the anonymity of the Internet.

Limitations of Verifying Digital Identities From a technical computer-engineering perspective, Friedman and Resnick

[8]For instance, eBay now has rules and policies on shilling which did not exist two years ago.

[9]eBay provides examples of questionable patters of shilling like "multiple bids by a bidder in short, deliberate intervals and bidding several times in small amounts even if not having been outbid" or "a member bids several times just under the highest bidder towards the end of a listing, incrementing the final sale price by a dollar and retracting if he/she inadvertently bids more than the high bidder."

(2001) propose the application of cryptographic techniques to bind online pseudonyms with true identities, which means that online users will apply digital signatures to make each web transaction truly accountable. This approach is feasible in theory but not in practice. The general public has not accepted cryptography as a common daily tool; most Internet users have not created public/private keys or used digital signatures to sign off emails and web transactions. Besides, the credible use of digital signatures requires reliable *public key infrastructure* (PKI), but current practices are far from meeting such a requirement[10]. Similarly, existing attempts of verifying digital identities, such as eBay's ID Verify, Microsoft Passport, or the Liberty Alliance Project[11], are limited in scope and, more importantly, subject to privacy issues and the intrinsic trust problem towards the authority itself.

Our Market Design Principle The current feebleness of anti-shilling detection and prevention methods in online auctions leaves room for fraud. We believe that proactive methods against this type of fraud are more effective than detective or reactive approaches, and designing incentive compatible trading mechanisms are as important as technical security implementations, especially when the feasibility of technical solutions is questionable. Klemperer (2002) states that "The most important features of an auction are its robustness against collusion and its attractiveness of potential bidders. Failure to attend to these issues can lead to disaster." Wilson (2001) has also emphasized that "one purpose of market design is to eliminate loopholes in the procedural rules that might be exploited by a wily trader." Therefore, we aim to proactively prevent shilling through market design.

III. Why can Shill Bidding Be Profitable in IPV English Auctions and How to Set the Optimal Shill Bid Level?

The Model We use an independent private-value (IPV) model assuming a single risk-neutral seller with valuation v_0 faces n risk-neutral bidders where bidder i holds valuation v_i, $i = 1, \ldots, n$, and the bidders' valuations are independent and identically distributed (IID) drawn from the common differentiable probability distribution $F(v)$.

To render tractable results, we assume that the auction is conducted as a conventional English thermometer auction but with the possibility of shilling:

- The seller pays the auctioneer a listing fee and discloses a reserve price to the auctioneer before the auction starts.
- The auctioneer controls a clock ticker starting at the reserve price and ascending continuously. Each bona fide bidder controls a "bid" button which lights up from the start of the auction, indicating he is active and bids at the ticker price. Once he turns off the light by pressing the button, he permanently quits the auction. During the auction, bidders cannot observe other bidders' buttons and do not know how many bidders are active.
- The seller controls a special "shill" button, which is initially inactive, but automatically lights up when only one bona fide bidder remains active. The seller can choose not to shill by immediately pressing her shill button or to shill up till a level by keeping her shill button lighted until the ticker price reaches that level.
- The auction ends when there remains only one active bidder (i.e., only one button is lighted), who will be the winner and pay the ticker price to the seller.
- The seller pays the auctioneer a commission fee.

Here we model the seller differently from other bidders because she knows who are the bona fide bidders. We assume that the ticker price at which only the highest bidder remains active is observable by the seller. This assumption easily holds for Amazon-type soft-close auctions but not for eBay-type hard-close auctions where many bidders bid only in the last minutes leaving no time for the seller to shill.

Profitability of Shilling Consider the typical auction model where the seller does not shill and there are no listing or commission fees. Let $U_n(r)$ denote the expected seller profit with n bidders and reserve price r. If the seller sells her item, she receives a payment equal to either the reserve or the second highest bidder valuation. The probability that the second highest valuation is at or below the valuation v is $F^n(v) + n(1 - F(v))F^{n-1}(v)$. Therefore, the seller's expected profit is

$$\begin{aligned} U_n(r) &= -(1 - F^n(r))v_0 + n(1 - F(r))F^{n-1}(r)r + \\ &\quad \int_r^\infty v \, \mathrm{d}\big(F^n(v) + n(1 - F(v))F^{n-1}(v)\big) \end{aligned}$$

Integrating by parts we get

$$\text{(1)} \quad \begin{aligned} U_n(r) &= -v_0(1 - F^n(r)) + \\ &\quad n \int_r^\infty (vF'(v) + F(v) - 1)F^{n-1}(v) \, \mathrm{d}v \end{aligned}$$

This is the same formula as that in Riley and Samuelson (1981) but with an extra $-v_0$ term because we consider expected profit rather than expected revenue.

Making the derivative of (1) zero yields the classical result of the optimal reserve r^* as that in Riley and Samuelson (1981).

$$\text{(2)} \qquad r^* = v_0 + \frac{1 - F(r^*)}{F'(r^*)}$$

Riley and Samuelson (1981) only focus on the special case where (2) has a unique solution and an ascending auction is optimal. But (2) may have multiple solutions corresponding to the

[10]PKI can only eliminate the use of false identities if there is a globally trusted authority that issues the keys to all users and verifies the true identity of the users before issuing a key. However, there is strong political opposition from privacy groups against such a central ID. There are also serious technical difficulties in implementing a global PKI infrastructure, as it is unclear how such a system can be protected against identity thefts.

[11]The Liberty Alliance Project is an alliance of more than 150 companies, non-profit and government organizations from around the globe. The consortium is committed to developing an open standard for federated network identity. http://www.projectliberty.org/.

local maxima and minima of the expected seller profit given by (1). When (2) has multiple solutions, $x-(1-F(x))/F'(x)$ is not monotone increasing in x and an ascending auction is no longer optimal (Myerson 1981). The reserve prices corresponding to the local maxima of the expected seller profit do not depend on n, but the expected profits corresponding to these reserves do. As n increases, the optimal reserve price r^{**} corresponding to the global maximum of the expected seller profit can shift from a smaller r_1^* to a larger r_2^*. Hence, the seller can ex ante set a smaller r_1^* and then optionally shill bid to reset a larger r_2^* based upon the additional information obtained from the bidding process.

(2) can have multiple solutions when $F(v)$ is a combination of value distributions of different types of bidders. When there are heterogeneous bidders, such as low-type vs. high-type, the expected seller profit may have multiple local maxima. Many auctions have heterogeneous bidders. For instance, in art and antique markets, experts and inexperienced collectors are different types of bidders. In California highway procurement auctions, there are bidders who already have got a large portion of their capacity committed and bid only for participation as well as bidders with little capacity committed who bid seriously for contracts (Jofre-Bonet and Pesendorfer 2000).

Before an auction, an estimation of the bidders' value distribution and how many bidders may participate can serve as a good predictor for the possible presence of a high-type bidder. During an auction, the seller can determine her shilling strategy conditional on her updated estimation of the possible presence of a high-type bidder based on the observed current high bid. Note that even if the seller knows exactly the number of bidders ex ante and thus knows that a larger solution of (2) globally maximizes her expected profit, the seller is still better off starting with a lower solution of the reserve to attract low-type bidders, and then pushing up the reserve to the larger solution via shilling when the current high bid reveals that there are possible high-type bidders.

An Example Figure 1 illustrates an example where the seller's optimal reserve corresponding to her global maximum expected profit changes with the number of bidders n and how shill bidding can be profitable. Assume the seller's valuation is \$20 and there are two types of bidders: 95% of the bidders draw their valuations from $N(\$20, \$20)$ and 5% of the bidders draw from $N(\$120, \$20)$, where $N(\mu, \sigma)$ denotes a normal distribution. Note that each of the expected profit curves, varying according to n, has two local maxima (peaks) corresponding to the same set of reserve prices. If $n < 12$, the expected seller profit at the first peak is higher than that at the second peak, corresponding to the smaller optimal reserve $r^{**} = r_1^* \approx \$38$. But if $n > 12$, the second peak is higher, corresponding to the larger optimal reserve $r^{**} = r_2^* \approx \$98$. When shilling is possible, the seller can set her ex ante optimal reserve price as \$38 assuming the worst scenario that only one low-type bidder would participate. However, during the auction, if the current high bid has passed \$50.4 (\$50.4 is the borderline reserve price where the expected seller profit starts to be lower than that of the second peak in the one-bidder case), then there is a high probability that a high-type bidder is present and the seller could shill bid up to her revised optimal reserve price \$98 corresponding to the second peak.

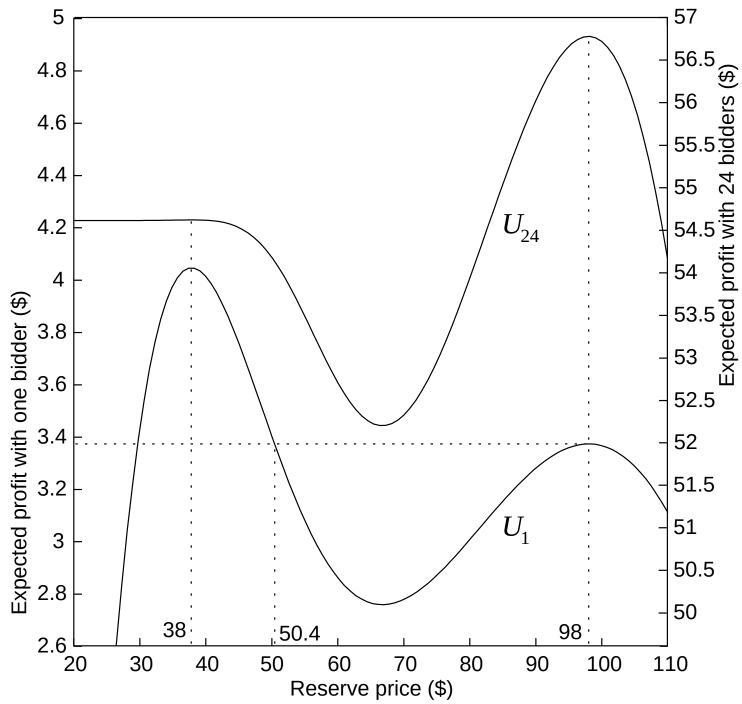

Figure 1. The seller's ex ante expected profits in an IPV English auction as a function of the reserve prices. The seller's valuation is $v_0 = \$20$, and bidders' value distribution is $0.95N(\$20, \$20) + 0.05N(\$120, \$20)$, where $N(\mu, \sigma)$ denotes a normal distribution. The seller's ex ante optimal reserve r^{**} is $r_1^* \approx \$38$ if there are less than 12 bidders, and is $r_2^* \approx \$98$ if there are more than 12 bidders. When there are 12 bidders, setting r^{**} at either \$38 or \$98 leads to the same expected seller profit. With the possibility of shilling, the seller's best strategy is to always ex ante set the reserve as \$38, then shill bid up to \$98 if the current high bid of the last remaining bidder has passed \$50.4.

The Optimal Shill Bid Level How does the seller know if her shilling will be profitable or not? If her shilling is profitable and she chooses to shill, how does the seller determine her optimal shill bid level to corner the remaining bona fide bidder facing the uncertainty about the valuation of her "rival"?

To calculate whether or not shilling is profitable and the optimal shill bid level, the seller has to calculate the Bayesian update of the last remaining bona fide bidder's value distribution. If the current high bid is b, then the seller knows that the remaining highest bidder's valuation v is drawn from the distribution F conditional on his valuation being at least b. Therefore, the updated cumulative distribution function of the remaining bidder, denoted by $G(v)$, is:

$$G(v) = F(v|v \geq b) = \frac{F(v) - F(b)}{1 - F(b)} \quad (3)$$

At this point, the seller has the option to shill. Her expected profit with a shill bid s, denoted by $\Pi(s)$, $s \geq b$, is

$$\Pi(s) = (s - v_0)(1 - G(s)) = \frac{(s - v_0)(1 - F(s))}{1 - F(b)} = \frac{U_1(s)}{1 - F(b)} \tag{4}$$

Because $s = b$ is equivalent to accepting the current high bid, $\Pi(b)$ represents the seller's expected profit if she chooses not to shill. Shilling is profitable if, for some $s > b$, $\Pi(s) > \Pi(b)$, which happens if and only if $U_1(s) > U_1(b)$.

The example in Figure 1 shows that the seller should ex ante set up a reserve price of \$38. When $\$50.4 < b < \$98$, it is profitable for the seller to submit shill bids up to \$98 because $U_1(b) < U_1(\$98)$. However, when $\$38 < b \leq \$50.4$, it is not profitable to shill.

Hence, with the possibility of shilling, the seller's best strategy is to ex ante set the optimal reserve price corresponding to the global maximum point of U_1 (the seller's expected profit when there is only one bidder), observe the bidding until one bidder remains, and submit shill bids up to the level corresponding to the global maximum of $\Pi(s)$ if the bidding goes high enough to indicate that there is a high probability that the remaining bidder is a high type. Note that $\Pi(s)$ is an affine scaling of U_1 truncated to the right of the current high bid b, therefore the global maximum of $\Pi(s)$ is also the global maximum of the truncated U_1. This is why the optimal shill bid level is the optimal reserve price corresponding to the truncated U_1. Consequently, there is no point setting the initial reserve price below the one corresponding to the global maximum of U_1 because the seller's optimal shilling strategy is to reach the global maximum of $\Pi(s)$, which requires the seller to always shill bid at least up to the reserve price corresponding to the global maximum of U_1.

The following theorem concludes the above arguments.

THEOREM 1: *In an IPV English auction with continuous bidding, shilling is profitable to the seller only if* $r = v_0 + \frac{1-F(r)}{F'(r)}$ *has a solution* $r^{*'} > r^*$, *where* $r^* = \arg\max U_1(r)$ *and* $r^{*'}$ *maximizes* $U_1(r)$ *for* $r \in [b, \infty)$ *where* b *is current high bid. The optimal shill bid level equals to* $r^{*'}$.

IV. What Is the Shill-Proof Fee (SPF) Schedule for English Auctions?

The SPF Schedule Our IPV English auction still uses the thermometer model. But our auctioneer applies a modified fee structure called the Shill-Proof Fee (SPF) schedule. Under the SPF, the seller pays the auctioneer a listing fee of $l(r)$ before the auction starts and a commission fee of $c(v-r)$ if the auctioned item is sold, where $c \in [0, 1)$ is the commission rate mathematically determined to ensure the non-profitability of shilling in a particular auction, v is the final sale price, and r is the seller's ex ante disclosed reserve price. $0 \leq l'(r) \leq c$ for all r and $l(r)$ is an increasing function.

Under the SPF, the seller is motivated to disclose her optimal reserve price ex ante rather than starting low and resetting it via shilling. If the seller discloses too low a reserve, she will be punished with a higher commission fee when the final sale price remains the same. If the seller discloses too high a reserve, she will be punished with a higher listing fee and a higher risk of no sale. The intricacies of the SPF work in a complementary fashion to encourage the seller to ex ante disclose her optimal reserve price close to her estimated final sale price based on her best knowledge about the valuation distribution of the bidders and the possible presence of a high-type bidder.

A. *How Does the SPF Work and How to Determine the Shill-Proof Commission Rate for an Auction?*

The Determination of a Shill-Proof Commission Rate To see how the SPF prevents shilling, we need to compare the expected seller profits with and without shilling and ensure that the seller's loss from shilling outweighs her possible gain.

As described in the English thermometer model, the seller can attentively observe the auction until only one bona fide bidder remains with the current high bid b. Let v be the valuation of this remaining bidder. At this point the seller can choose either not to shill with a gain of (Π_c denotes the seller expected profit under the SPF with the commission rate c)

$$\Pi_c(b) = b - v_0 - l(r) - c(b - r)$$

or to pick a shill bid level s, $s > b$, and let the clock ticker increase until either the remaining bona fide bidder quits or the clock reaches s. The former happens when $v < s$, and the shill bid wins the item at price v. The seller retains the item and has a loss equal to the intermediation fees $\Pi'_c(v) = -l(r) - c(v - r)$. If the clock ticker reaches s, the remaining bidder outbids the seller's shill bid and pays s. The seller loses the listing fee $l(r)$ and the item v_0, but receives a payment of $s - c(s - r)$. Therefore the seller's gain is $\Pi''_c(s) = s - v_0 - l(r) - c(s - r)$. Recall that $G(v)$ in (3) represents the conditional probability that the winning bid $\in [b, v)$. Hence, $G(s)$ is the conditional probability that the shill bid s is not outbid. Combining the above two scenarios and integrating by parts, the expected seller profit from shilling is

$$\begin{aligned} \Pi_c(s) &= \int_b^s \Pi'_c(v)\,\mathrm{d}G(v) + \Pi''_c(s)(1 - G(s)) \\ &= (1 - G(s))(s - v_0) - \\ &\quad l(r) - c\Big[s - r - \int_b^s G(v)\,\mathrm{d}v\Big] \end{aligned} \tag{5}$$

Note that $\Pi_0(s)$ (no commission case) is the same as $\Pi(s)$ in (4).

A shill-proof commission rate c needs to satisfy $\Pi_c(s) \leq \Pi_c(b)$, that is,

$$(1 - G(s))(s - v_0) - (b - v_0) \leq c \int_b^s (1 - G(v))\,\mathrm{d}v$$

Replacing $G(s)$ with $\frac{F(s)-F(b)}{1-F(b)}$ gives

(6) $$c \geq \frac{(1-F(s))(s-v_0)-(1-F(b))(b-v_0)}{\displaystyle\int_b^s (1-F(v))\,\mathrm{d}v}$$

Inequality (6) yields the lower bound of a commission rate for an auction above which shilling is non-profitable. The upper bound of the commission rate can be determined by other factors such as the competition among auctioneers, because too high a commission rate can make an auctioneer less competitive.

(6) also shows that a commission rate, c, may vary across different auctions, and is mathematically related to the bidders' value distribution unique to an auction. An auctioneer should charge a higher commission rate in an auction where shilling is more likely to be a problem than in an auction where shilling has less impacts.

Note that the commission rate in (6) does not depend on the listing fee. The listing fee must be paid before the auction starts, so it is not related to shilling.

Simplification Determining the lower bound of a shill-proof commission rate using (6) is complicated. Next we simplify its calculation. We start with a special case, then prove that its result can be generalized.

Consider one special case when s is a timid shill bid, only slightly above the current high bid b. Calculating the limit of (6) as $s \downarrow b$ gives

(7) $$c \geq 1 - \frac{b-v_0}{1-F(b)}F'(b)$$

(7) represents the lower bound of a commission rate under which a timid shill bid is not profitable.

The following theorem shows that if a commission rate is chosen to discourage timid shill bids according to (7), then such a commission rate also ensures that no other shill bid, timid or bold, is profitable.

THEOREM 2: *In an IPV English auction with continuous bidding and under the SPF schedule, shill bidding is not profitable regardless the current high bid b if and only if the commission rate $c \geq 1 - \frac{b-v_0}{1-F(b)}F'(b)$ holds for all $b \geq r$, where r is the ex ante reserve price.*

PROOF. By contradiction. Assume that (7) holds for all $b \geq r$ but shilling is profitable, i.e., $\exists s$, $s > b$, s.t., $\Pi_c(s) > \Pi_c(b)$. This implies that $\exists \xi$, $b < \xi < s$, s.t., $\Pi_c'(\xi) > 0$. Taking the derivative of (5) and using $F(b) \leq F(\xi)$, we get

$$\begin{aligned} 0 < \Pi_c'(\xi) &= (1-c)\frac{1-F(\xi)}{1-F(b)} - \frac{\xi - v_0}{1-F(b)}F'(\xi) \\ &\leq (1-c) - \frac{\xi - v_0}{1-F(\xi)}F'(\xi) \end{aligned}$$

which is equivalent to

$$c < 1 - \frac{\xi - v_0}{1-F(\xi)}F'(\xi)$$

This contradicts our assumption. On the other hand, if (7) does not hold for some b, then for some sufficiently small $\varepsilon > 0$, a timid shill bid of $s = b + \varepsilon$ is profitable. ■

According to Theorem 2, we can use inequality (7) as a simplified calculation of the lower bound of a commission rate that prevents all shill bids, timid or bold. A chosen commission rate for an auction should be the maximum of the right-hand side of (7) for all b, $b \geq r$ where r is the seller's disclosed ex ante reserve.

Note that the sign of the right-hand side of (7) is the same as the sign of the first derivative of the expected seller profit function with no commission U_n in (1). It is interesting to see the similarity between the right-hand side of (7) and the optimal reserve price satisfying $(r^* - v_0)\frac{F'(r^*)}{1-F(r^*)} = 1$ (a transformation of (2)). Note that when $b = r^*$ the right-hand side of (7) is zero, indicating that when the current high bid equals to the optimal reserve in a no-commission auction, no shill bid is profitable. In addition, for most distributions, $1-(b-v_0)\frac{F'(b)}{1-F(b)}$ permanently stays below zero after some large enough b. For example, the minimum commission rate shown in Figure 2 stays negative for $b > \$98$. This simply implies that a shill bid far in the right tail of the bidders' value distribution will never be profitable even in a no-commission auction.

Examples Figure 2 shows the minimum commission rate the auctioneer should charge to prevent shilling in the auction where the bidders' value distribution is $0.95N(\$20, \$20) + 0.05N(\$120, \$20)$ and the seller's valuation v_0 is \$20. Even without any commission, a shill bid is not profitable if $b \in [\$38, \$50.4]$, because, in this range, the expected seller profit is higher than the second peak in the one-bidder case depicted in Figure 1. When $b \in [\$50.4, \$66.70]$, a timid shill bid is not profitable but a bold one of \$98 is (\$66.70 is the current high bid level above which timid shill bids become profitable, i.e., where the minimum commission rate in Figure 2 becomes positive). A timid shill bid is not profitable in this range because it is more likely to cause low-type bidders to drop out, and less likely to push the payment of a possible high-type bidder high enough to compensate for this loss. However, a bold shill bid of \$98 is profitable in this range because it is large enough to possibly extract, from a possibly present high-type bidder, a payment that is high enough to compensate for the decreased probability of successful shill bidding. When $b \in [\$66.70, \$98)$, any shill bid, timid or bold, is profitable. No matter what b is, as long as the auctioneer sets the commission rate above 60% (the maximum of all the minimum c for all possible b, $b \geq$ the seller's disclosed ex ante reserve), no shill bid, timid or bold, is profitable. 60% is kind of high because Figure 2 depicts a rather extreme case where the mean valuations of two types of bidders are quite apart.

Take another example where the two types of bidders are not quite apart. If the bidders' value distribution is $0.95N(\$8700, \$400) + 0.05N(\$10400, \$400)$ and the seller's valuation is \$8700, the commission rate required to prevent

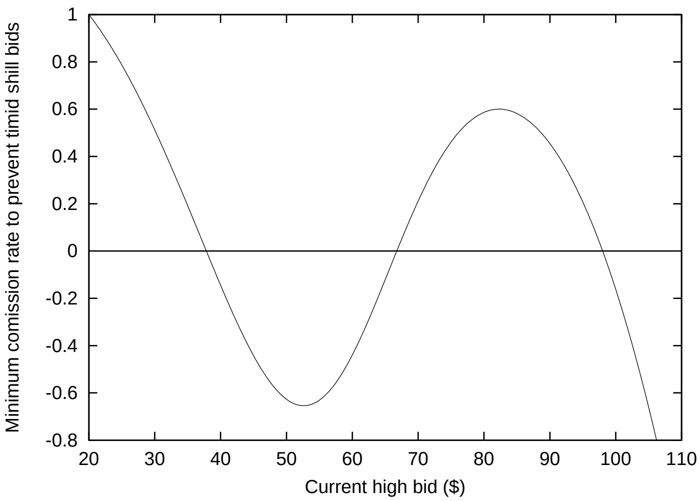

Figure 2. The minimum commission rate c (as a function of the current high bid b) that the auctioneer should charge to make shill bids slightly above the current high bid unprofitable under the SPF. Assume the bidders' value distribution is $0.95N(\$20, \$20) + 0.05N(\$120, \$20)$ and the seller's valuation $v_0 = \$20$. A negative c here means that for a given high bid, the expected seller profit would decrease if she uses a timid shill bid. Taking the maximum of this curve truncated to the right of the optimal ex ante reserve price provides the commission rate which makes all shill bids unprofitable.

shilling is only 16.9%. With such a commission rate and an assumption of ten bidders, the seller's optimal reserve (how to compute it is shown in the next section) should be set at $\$9147.50$ if there is no listing fee. Introducing a 0.5% listing fee, which is comparable to that in eBay, would slightly reduce the optimal reserve to $\$9146.80$. If the final sale price is \$10,000, the commission fee is only \$144, similar to the commission fee of \$163.12 currently charged by eBay. This shows that for honest sellers, an English auction with the SPF is no more expensive than current eBay auctions. However, for sellers starting with a low reserve, say, \$5000, the SPF commission fee would be \$845, much higher than that in eBay.

Practicality Adopting the SPF does not affect honest sellers and bidders. An honest seller can minimize her commission fee because the SPF rewards the truthful disclosure of her ex ante optimal reserve. In case the seller's sale price is way above her reserve, the seller would probably not mind paying higher commission fee out of her surprisingly high profit. Bidders with valuations above the reserve are not affected by the change in the seller's intermediation fees. Bidders with valuations below the reserve would not join the auction; they are less desirable by the seller anyway. Under the SPF, a bidder's best strategy is still simple, i.e., to bid up to his valuation.

The requirement to choose a commission rate that is shill-proof may be relaxed. It is possible that a shill-proof commission rate would be too high to be practical or acceptable by sellers, especially in auctions where shill bidding is rare. Even when heterogeneous bidders exist in an auction and shilling can be profitable, it is possible that the seller in the auction is ethical and does not conduct such a fraud. Then it is not necessary to have a very restrictive commission rate to prevent a very unlikely shill bid. Therefore, we can also regard **SPF%** as a *shill-protection-factor*, indicating an auctioneer's tolerance level of shilling in an auction, that is, he can choose a commission rate that is not shill-proof but ensures that the probability of profitable shilling is below $(1-\text{SPF}\%)$. For instance, a commission rate with SPF95 means that the probability of profitable shilling is below 5%. A shill-proof commission rate calculated by (7) has a factor of SPF100. How an auctioneer calculates the SPF% factor of his current commission rate and how to determine a commission rate satisfying a particular SPF% are future research issues outside the scope of this paper.

B. *How Does the Seller Set Her Ex Ante Optimal Reserve Price(s) under the SPF?*

In an English auction under the SPF schedule, the seller still needs to set her ex ante optimal reserve price that maximizes her expected profit. The expected seller profit under the SPF is denoted by $U_n(r, c)$, which is a function of the number of bidders, reserve price, and the commission rate for the particular auction that the seller will enter.

Note that the conventional English auction without the listing and commission fees is a special case of the English auction under the SPF with $c = 0$. This implies that $U_n(r, 0) = U_n(r)$. We rewrite (1) as

$$\text{(8)} \quad U_n(r,0) = (r - v_0)(1 - F^n(r)) - r(1 - F^n(r)) + n\int_r^{\infty} (vF'(v) + F(v) - 1)F^{n-1}(v)\,\mathrm{d}v$$

By rewriting, we divide the expected seller profit into two parts. The first term in (8) is the expected seller profit from the payment equal to the reserve (including the case where the item is not sold). The remaining two terms represent the extra expected profit from the payment above the reserve.

To express $U_n(r, c)$, the first term in (8) does not change because there is no commission fee if the seller is paid only at the reserve, but the remaining two terms have to be multiplied by $(1 - c)$ because the seller only gets $(1 - c)$ portion of the sale price above the reserve. Merging the first two terms and subtracting the listing fee we get:

$$\text{(9)} \quad U_n(r,c) = (cr - v_0)(1 - F^n(r)) - l(r) + (1-c)n\int_r^{\infty} (vF'(v) + F(v) - 1)F^{n-1}(v)\,\mathrm{d}v$$

We can rearrange (9) as

$$\text{(10)} \quad U_n(r,c) = c(r - v_0)(1 - F^n(r)) - (1-c)v_0(1 - F^n(r)) - l(r) + (1-c)n\int_r^{\infty} (vF'(v) + F(v) - 1)F^{n-1}(v)\,\mathrm{d}v$$

Note that, without considering the listing fee, the seller's expected profit in an English auction under the SPF is a convex combination of the expected seller profits in the all-bidder-collude auction (Graham and Marshall 1987) with weight c and that in the no-bidder-collude auction (Riley and Samuelson 1981) with weight $(1-c)$. If all bidders collude, they agree that only one bidder bids at the reserve and therefore, the expected seller profit is $(r - v_0)(1 - F^n(r))$. If no bidder colludes, the expected seller profit is given by (1).

To obtain the seller's optimal reserve under the SPF, differentiating (9) with respect to r we get

$$\frac{\partial U_n}{\partial r}(r,c) = c(1-F^n(r)) - l'(r) - nF^{n-1}(r)[F'(r)(r-v_0) - (1-c)(1-F(r))]$$

Assuming $F'(r) > 0$, we can rewrite the above as

$$(11)\quad \frac{\frac{\partial U_n}{\partial r}(r,c)}{nF^{n-1}(r)F'(r)} = = v_0 + (1-c)\frac{1-F(r)}{F'(r)} + c\frac{1-F^n(r) - \frac{l'(r)}{c}}{nF^{n-1}(r)F'(r)} - r = v_0 + (1-c)\frac{1-F(r)}{F'(r)} + c\frac{1-F^n(r) - \frac{l'(r)}{c}}{F^{n\prime}(r)} - r$$

At the optimal reserve, equation (11) is zero and it changes sign from positive to negative. Thus the optimal reserve satisfies

$$(12)\quad r^* = v_0 + (1-c)\frac{1-F(r^*)}{F'(r^*)} + c\frac{1-F^n(r^*) - \frac{l'(r^*)}{c}}{nF^{n-1}(r^*)F'(r^*)} = v_0 + (1-c)\frac{1-F(r^*)}{F'(r^*)} + c\frac{1-F^n(r^*) - \frac{l'(r^*)}{c}}{F^{n\prime}(r^*)}$$

Again, interestingly, the seller's ex ante optimal reserve under the SPF is a convex combination of the optimal reserve price in the all-bidder-collude case with weight c and that of the no-bidder-collude case with weight $(1-c)$.

When $c = 0$ and excluding the consideration of the listing fee, (12) becomes (2), the result in the classical auctions. Also note that if there is only one bidder, i.e., $n = 1$, (12) becomes (2) as well because the final sale price equals to the reserve.

As we have discussed earlier, the listing fee in the SPF prevents the seller from setting too high a reserve. We can see from (12) that charging a listing fee does reduce the optimal reserve price. We can also obtain the upper bound of $l(r^*)$ ensuring $U_n(r^*, c) > 0$.

An Example In our example where the bidders' value distribution is $0.95N(\$20,\$20) + 0.05N(\$120,\$20)$ and the seller's valuation v_0 is \$20, with a 60% commission rate and a 0.5% listing free, the seller's optimal ex ante reserve price varies between \$37.35 and \$41.20 for four or fewer bidders, and is \$98.21 or higher for five or more bidders.

V. Concluding Remarks

We believe that the rise of online auction fraud is largely caused by the uniqueness of auctioning over the Internet, that is, dynamic pricing determination in a virtual environment where traders' identities are easy to disguise. The dynamic pricing determination offers flexibility but also leaves room for unlawful manipulation that is hard to detect. The virtual environment has an intrinsic problem – the anonymity of trading – which creates new venues for fraud like shill bidding.

Interestingly, Klemperer (2002) has recommended that bids be made anonymously as a method of making the English auction more robust against bidder collusion, however, anonymous bids, which have become widespread in online auctions, have made the English auction vulnerable to seller self-collusion in the form of shilling.

We believe that the most effective method against fraud is to design incentive-compatible mechanisms and policies. A similar principle is demonstrated in suggestions to solve the anonymity problem of the Internet. Friedman and Resnick (2001) recommend charging for name changes to discourage the creation of multiple aliases and mitigate the inherent social cost of free name changes. Dellarocas (2003) suggests that "the design that results in optimal social efficiency is one where the mechanism sets the initial profile of new members to correspond to the 'worst' possible reputation." Rather than solving the general anonymity problem using incentive alignment, Wang, Hidvégi and Whinston (2001–2002a) focus on the design of an auction protocol itself, suggesting a new mechanism for multi-unit sealed-bid auctions that is robust against bidders' false-name bids. Yokoo, Sakurai and Matsubara (2004) generalizes the issue of bidders' false-name bids and show one sufficient condition where the Vickrey-Clarke-Grove mechanism is false-name-proof. In this paper we redesign the policies related to an auction instead of the auction mechanism itself to fight against the seller's false-name bidding.

Applying the SPF in English auctions discourages shilling behavior from rational sellers. On the other hand, it does not add cumbersome rules that restrict honest sellers' and bidders' flexibility, and only requires changes on the auctioneer's side. Our SPF design indicates that the policies of a market maker are essential to ensuring the integrity of a market and to enhancing the trust among traders.

The following research directions are of interest and they also indicate the limitations of the paper. First, our auction model assumes IPV bidders, instead of common-value or affiliated-value bidders (Milgrom and Weber 1982). Shilling does have severe negative impacts in common-value auctions (Chakraborty and Kosmopoulou 2004, Kauffman and Wood 2005). Shilling affects how bidders update their valuations upon the information aggregated from others' bids, because bidders can no longer assume that behind every bid there is a bona fide bidder willing to pay that price.

Shill bidding also has similar negative effects on IPV multiple-round English auctions. Preliminary study in Wang, Hidvégi and Whinston (2002b) extends McAfee and Vincent (1997)'s model and shows that shill bidding, or even the perception of it, destroys the seller's ability to enforce any reserve price in multiple-round English auctions.

Even though we have not yet theoretically studied the SPF in other auction models, we believe that the SPF can reduce the negative impacts of shilling in other settings as well because the intuition remains the same – the SPF provides an incentive for the seller to choose a reserve price close to the real value of the item, and hence increases bidders' trust in the market. We would also like to study how the SPF enhances traders' trust in an auction market as a whole, including the integrity of the market reputation system.

Last but not least, we could further extend our study on the shill-protection-factor SPF%.

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